The physical parameters appearing in the similarity criteria and the formulas have the following meaning and dimensions: c, specific heat, $[L]^2[T]^{-2}[\Theta]^{-1}$; R, stress, compressive or tensile strength, or shear stress, $[M][L]^{-1}[T]^{-2}$; G, shear modulus, $[M][L]^{-1}[T]^{-2}$; ρ, density, $[M][L]^{-3}$; λ, thermal conductivity, $[M][L][T]^{-3}[\Theta]^{-1}$; g, acceleration due to gravity, $[L][T]^{-2}$; K, elasticity coefficient, $[M][L]^{-1}[T]^{-2}$; E, Young's modulus, $[M][L]^{-1}[T]^{-2}$; μ, Poisson's ratio; b, a parameter, $[L]^{1/2}$; a, a coefficient, $[M]^{-n}[L]^{n-1/2}[T]^{2n}$; n, a power factor; σ, stress or pressure, $[M][L]^{-1}[T]^{-2}$; γ, specific weight, $[M][L]^{-2}[T]^{-2}$; ε, relative deformation; k, rigidity, $[M][T]^{-2}$; Q, heat transmitted through the surface normal to the wall in the direction of decrease per unit time, $[M][L]^2[T]^{-2}$; F, area, $[L]^2$; δ, wall thickness; [L]; t_1-t_2, temperature difference between opposite surfaces of the wall; °C, $[\Theta]$; R_1, grade of concrete, $[M][L]^{-1}[T]^{-2}$; R_c, cement activity, $[M][L]^{-1}[T]^{-2}$; C, cement mass, [M]; W, water mass, [M]; E_I, initial elasticity modulus of concrete under compression and tension, $[M][L]^{-1}[T]^{-2}$; α, an index, $[M][L]^{-1}[T]^{-1}$. The symbols in square brackets denote dimensions in SI units: [M], mass, kg; [L], length, m; [T], time, sec; $[\Theta]$, temperature, deg.

Thus, the equations proposed – Eqs. (9), (12), (15), and (16) in general form and Eqs. (13), (14), (17), and (18) for heavy concretes – characterizing the functional relations between a series of physicomechanical parameters of both mixtures and artificial constructional conglomerates, may be used to determine the basic physicomechanical and thermotechnical parameters of ACC and their mixtures.

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USE OF FINITE-PENETRATION-DEPTH METHOD TO CALCULATE THE HEATING OF A PLANE PLATE UNDER THE ACTION OF A RADIANT HEAT FLUX

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Using the finite-penetration-depth method, a solution is obtained to the problem of plate heating a radiant flux. The results are compared with a numerical solution.

The heat-conduction problem with Stefan–Boltzmann boundary conditions is of considerable difficulty for analytic consideration, and requires linearization of the boundary conditions, or the use of numercial methods [1].

Below, the solution of one problem of this type by the finite-penetration-depth method [2, 3], an analog of the integral methods of boundary-linear theory, is considered. The basic idea is that it may be assumed, with sufficient accuracy for practical purposes, that heat penetrates into a heated body only to a finite depth, which is known as the heated layer. Following [2, 3], the heat-conduction equation for an infinite plane plate

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$$\frac{\partial t}{\partial \tau} = a \frac{\partial^2 t}{\partial x^2} \tag{1}$$

is considered, with the initial condition $t = t_0$. Assume that at the boundary of the heated layer the following condition is satisfied

$$t|_{x=\Delta} = t_0, \quad \frac{\partial t}{\partial x}\bigg|_{x=\Delta} = 0 . \tag{2}$$

Integrating Eq. (1) with respect to x from 0 to Δ, it is found that

$$\frac{d}{d\tau}\left[\int_0^\Delta t dx - t_0\Delta\right] = a \frac{\partial t}{\partial x}\bigg|_0^\Delta . \tag{3}$$

It is assumed that the temperature distribution inside the heated layer may be approximated by the quadratic parabola

$$t = t_0 + \frac{q_0}{2\lambda\Delta}(x - \Delta)^2 , \tag{4}$$

where q_0 is the heat flux at the plane surface.

Using Eqs. (2) and (4), Eq. (3) may be brought to the form [3]

$$\frac{aq_0}{\lambda} = \frac{d}{d\tau}\left[\frac{q_0\Delta^2}{6\lambda}\right] . \tag{5}$$

Now consider the case when the heat flux q_0 at the plate surface is given by the Stefan–Boltzmann law

$$q_0 = 4.9\varepsilon\left[\left(\frac{T_m}{100}\right)^4 - \left(\frac{T_0 + \frac{q_0\Delta}{2\lambda}}{100}\right)^4\right] , \tag{6}$$

where T_m is the absolute radiation temperature; $T_0 = t_0 + 273°$; $T_0 + q_0\Delta/2\lambda$ is the absolute surface temperature $(x = 0)$ according to Eq. (4).

The following expression for Δ is obtained from Eq. (6)

$$\Delta = \left\{100\left[\left(\frac{T_m}{100}\right)^4 - \frac{q_0}{4.9\varepsilon_{re}}\right]^{1/4} - T_0\right\}\frac{2\lambda}{q_0} . \tag{7}$$

Substituting this expression into Eq. (5) yields

$$\frac{3}{2}\frac{aq_0}{\lambda^2} = \frac{d}{d\tau}\frac{\left\{100\left[\left(\frac{T_m}{100}\right)^4 - \frac{q_0}{4.9\varepsilon_{re}}\right]^{1/4} - T_0\right\}^2}{q_0} . \tag{8}$$

The following notation is introduced

$$y = q_0/4.9\varepsilon_{re}\left(\frac{T_m}{100}\right)^4 , \quad \beta = \frac{T_0}{T_m} ,$$

$$z = \frac{3a\left[4.9\varepsilon_{re}\left(\frac{T_m}{100}\right)^4\right]}{2\lambda^2 T_m^2}\tau .$$

The quantity y characterizes the change in heat flux over time. Below, it is the change in this quantity which will mainly be of interest.

In the dimensionless variables y, z, Eq. (8) takes the form

$$y = \frac{d}{dz}\frac{[(1 - y)^{1/4} - \beta]^2}{y} \tag{9}$$

with the initial condition

$$y = 1 - \beta^4 \text{ when } z = 0. \tag{10}$$

There exists an analytic solution to Eq. (9), though it is somewhat cumbersome in form

$$z = \frac{1}{8}\ln\frac{1+\sqrt{1-y}}{1-\sqrt{1-y}} + \frac{1}{4}\sqrt{1-y}\,\frac{2+y}{y^2} - $$
$$-2\beta\left[\frac{(1-y)^{1/4}}{2y^2} + \frac{(1-y)^{1/4}}{8y} + \frac{3}{32}\ln\frac{1+(1-y)^{1/4}}{1-(1-y)^{1/4}} + \frac{3}{16}\,\mathrm{arctg}\,(1-y)^{1/4}\right] + \frac{\beta^2}{2y^2} + C, \tag{11}$$

where C is a constant of integration

$$C = 2\beta\left[\frac{\beta}{2(1-\beta^4)^2} + \frac{\beta}{8(1-\beta^4)} + \frac{3}{32}\ln\frac{1+\beta}{1-\beta} + \right.$$
$$\left. + \frac{3}{16}\,\mathrm{arctg}\,\beta\right] - \frac{\beta^2}{2(1-\beta^4)^2} - \frac{1}{8}\ln\frac{1+\beta^2}{1-\beta^2} - \frac{\beta^2}{4}\,\frac{3-\beta^4}{(1-\beta^4)^2}. \tag{12}$$

Because it is so cumbersome, Eq. (11) is not suitable for practical calculations. To obtain the solution in more expedient form, the two limiting cases may be considered.

A. The Case of Small z. Here Eq. (9) may be written, taking Eq. (10) into account, in the approximate form

$$1 - \beta^4 = \frac{d}{dz}\,\frac{[(1-y)^{1/4} - \beta]^2}{1-\beta^4}, \tag{13}$$

which may be integrated to give

$$y = 1 - [\beta + (1-\beta^4)\sqrt{z}]^4. \tag{14}$$

This solution satisfies the initial condition in Eq. (10);

B. The Case of Small y (large z). In the case the relation $(1-y)^n \simeq 1-ny$ is used and, in addition, it may be shown that Eq. (12) can be approximated with high accuracy by the simple formula

$$C = \beta^2/2. \tag{15}$$

The solution in Eq. (11) then simplifies

$$y = \frac{2(1-\beta)}{\sqrt{8z + 1 - \frac{\beta}{2} - 4\beta^2}}. \tag{16}$$

In Fig. 1, curves of y(z) plotted from Eqs. (14) and (16) for $\beta = 0$, 0.25 are shown. The most expedient interpolation between the curves of Eqs. (14) and (16) for intermediate values of z is given by the tangent to these two curves. Thus, for given β, it is sufficient, in order to construct a dimensionless heating curve of the outer surface, to plot curves according to Eqs. (14) and (16) and draw their common tangent. It is more expedient to find the equation of the tangent for each specific problem than in the general case.

In [4], the results of numerical solution of the problem of the heating of a plate by a radiant heat flux conduct through the plane x = 0, with the condition of thermoinsulation ($\partial T/\partial x = 0$) at the plane x = h, are given in nomogram form. The parameters of the nomograms are the dimensionless complexes $\beta = T_0/T_m$; Fo = $a\tau/$

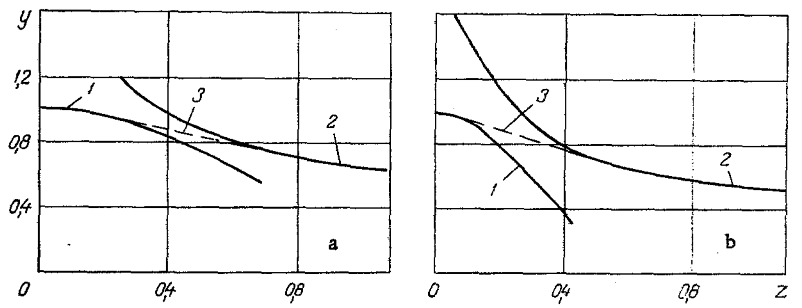

Fig. 1. Curves of y as a function of z for $\beta = 0$ (a) and 0.25 (b): 1) from Eq. (14); 2) from Eq. (16); 3) linear interpolation.

TABLE 1. Comparison of Accurate and Approximate Solutions

β	Fo	Bo	θ_{acc}	θ_{approx}
0	0,1	1,0	0,36	0,387
0	0,5	1,0	0,72	0,728
0	1,0	1,0	0,87	0,862
0	0,1	0,5	0,65	0,642
0	0,5	0,5	0,89	0,88
0	0,1	2,0	0,275	0,274
0	1,0	2,0	0,62	0,72
0,25	0,1	0,5	0,70	0,70
0,25	0,5	0,5	0,90	0,884
0,25	0,1	2,0	0,26	0,337
0,25	1,0	2,0	0,728	0,72

h^2; $Bo = \lambda/(4.9 \cdot 10^{-8})\varepsilon_{re}T^3h$. According to the idea of the finite-depth method, the formulas given are valid only for $\Delta \leq h$, which corresponds to $Fo < a\tau/na\tau = 1/n$, where $n \sim 5$–10 (when q = const, n = 6). However, if is evident from a comparison with the accurate solution that the formulas obtained give good agreement up to Fo = 1. Table 1 compares the approximate and accurate values of the surface temperature $\theta = (T_{sur} - T_0)/(T_m - T_0)$. As is evident, the agreement between them should be regarded as good. Note that the "accurate" values are taken from nomograms, which may lead to pronounced errors in determining θ, and hence also the heat flux at the surface. Interpolation in β is especially inconvenient when using nomograms. In some cases, therefore, the approximate formulas obtained may be found more expedient for practical calculations than a numerical solution in the form of nomograms.

NOTATION

t, temperature; T, absolute temperature; λ, a, thermal conductivity and diffusivity of plate material; ε_{re}, reduced emissivity; x, coordinate; q, specific heat flux; τ, time.

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